INFLUENCE OF INELASTIC BEHAVIOR ON DYNAMIC RESPONSE

by

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INTRODUCTION

The inclusion of the effects of inelastic behavior has been a fairly recent development in the analysis and design of structures. It has been a most welcome development in that inelasticity is a more realistic assessment of real material properties; on the other hand, it also introduces a much more complex situation in terms of analysis and the application of the analysis to the development of realistic design procedures.

The development of methods of plastic design and limit design is fairly advanced for static structures but the application of such procedures to dynamic problems is at a more primitive stage of development. Just as inelasticity complicates static analysis, the consideration of the effect of inelasticity in the dynamic state creates a complexity of a higher order of magnitude.

It is the purpose of this lecture to explore this field and to observe some of the more pertinent developments without delving into the mathematical details. The various types of inelasticity will be introduced and the more common types discussed in some detail. Mathematical models of the most common structures will be described, making us of the basic inelastic properties of the structural components. The basic characteristics of a number of the methods of analysis will be described in order to illustrate the varied approaches which can be used to solve the same basic problem. A description of the general effects of inelasticity will be given based upon the observations of a number of investigators in this field. An alphabetical list of references will enable the interested engineer to examine in detail any of the material presented in this lecture by referring to the appropriate publication.

TYPES OF INELASTICITY

As a review of the basic strength of materials, consider first the stress-strain diagram for mild steel as shown in Figure 1. Figure 1(a) illustrates the stress-strain curve to failure, with the appropriate portions labelled elastic, plastic and strain-hardening. Figure 1 (b)

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shows the details near the yield-point. In particular, one should note that the yield-point is defined as the stress at which plastic flow begins and full rebound is no longer possible. Another point on the curve, usually defined as the proportional limit, is the stress after which stress is no longer linearly proportional to strain. It is often said that the stress-strain curve is non-linear after this point although it remains elastic for a short portion of the curve above this point. It is important to realize the difference between the onset of non-linearity and the onset of inelasticity, although they are often at or near the same position on the stress-strain curve.

After this review, it is now possible to define the typical types of inealsticity used in dynamic response analysis. Figure 2 shows the common types, as well as the elastic case. All types are of necessity somewhat idealized but compared to the assumption of purely elastic behavior, they are an improvement in the representation of the physical material properties. Figure 2 (a) illustrates the linear elastic stress-strain relationship, which is the common basis for almost all structural analysis and for the theory of elasticity. Figure 2 (b) illustrates the most common inelastic idealization, the so-called elastic-perfectly plastic material. It is seen that this is reasonably representative of mild steel for a much larger range of strains than the normal elastic representation. Figure 2(c) shows a modification of this in which the elastic portion of the curve is neglected and which is known as a rigid-plastic material. This idealization is often used to assess the effects of very large plastic deformation, for which case the elastic portion of the curve is of little consequence. Other materials are often best described by the elastic-plastic strain hardening model of Figure 2 (d). Many other idealizations are possible but these are the common types used to represent structural elements. Figures 2 (e) and (f) illustrate two types of characteristics which are possible for non-structural components of framed buildings. The inclusion of such elements when calculating the resistance of a frame depends upon the construction of the individual building; discussions of this matter are included in several of the papers in the list of references (see references B10, B11, B12, B13 and N1).

For members subjected to axial loads, the stress-strain curves are directly proportional to the load-deformation curves. For members subjected to bending, the deformation characteristics of an element are expressed by the moment-curvature relationship, which itself is a function of the shape of the member as well as of the stress-strain characteristics of the material. Figure 3 (a) illustrates typical curves computed for elastic-perfectly plastic materials. Figure 3 (b) shows the idealizations of these characteristics which are commonly used in analysis. The so-called perfectly plastic case (in which the moment after yield is considered constant) is the usual assumption. Note that this implies the use of thin-walled members such as I beams or wide glange beams. If solid members are being used, care must be taken to use the appropriate bilinear approximation.

Consider a single degree-of-freedom system having the elastic-perfectly plastic force-deformation characteristic shown in Figure 4. The spring deformation at yield is denoted by U_y and the maximum deformation attained during any dynamic load application is denoted by U_m. A measure of the amount of inelastic deformation is given by the ductility factor μ which is defined as:

$$\text{ductility factor } \mu = \frac{U_m}{U_y}.$$

Another measure of inelastic action is given by the reduction factor, which is defined as the ratio of the yield deformation U_y over the peak elastic deformation U_e. The ductility factor times the reduction factor gives the ratio of maximum inelastic deformation to the maximum elastic deformation. These measures will be referred to later in the lecture and are used extensively when describing the effects of inelasticity.

MATHEMATICAL MODELS USED IN ANALYSIS

Consider first a simple single story rigid frame as shown in Figure 5 (a). Dynamically, this is usually considered as a single degree-of-freedom system with all the mass considered to be concentrated at the floor. When determining the structural resistance of this frame, two possible mathematical models may be used. Figure 5 (b) shows the most common, in which the floor is considered to be rigid and having motion parallel to the foundation. The supporting members are symmetrical so that all four plastic hinges will form at the same time. This model is then identical to the single mass oscillator shown in Figure 4, with the equivalent spring force being the shear in the columns. A second, more general model is useful when considering non-symmetrical situations and the inclusion of the effects of static vertical floor loadings on the dynamic response. This model is shown in Figure 5 (c) and permits plastic hinges to form in any order as the dynamic response dictates. The columns may have different stiffnesses and yield moments; hinges are also allowed to form under any concentrated vertical static loads if the beam yield moment is reached at these points during the dynamic response. This second model is also a single degree-of-freedom system but the equivalent spring force in this case is a piece-wise linear curve (as shown) in which the actual curve is a function of the dynamic and static loading in addition to the properties of the basic frame. Viscous damping can be included in both models if desired.

The models used for multi-story framed structures are in most cases extensions of the two models used for the single storey frame. The simplest model is the so-called shear building in which all floor systems are assumed rigid (insofar as dynamic motion is concerned). All floors move parallel to each other so aonly inter-floor shear need be considered. This inter-floor shear is assumed to have an elastic-perfectly plastic force-deformation relationship. Ordinarily, the story-to-story yield variation is assumed to be proportional to an inverted triangle. Other variations are also possible. (See references B10 and P1.)

The next mathematical model is the frame in which all members are flexible and plastic hinges are allowed to form both in columns and beams. This model is generally known as the elasto-plastic rigid frame. Some methods of analysis employing this model limit hinge formation to certain pre-designated pairs; whereas, others allow more complete freedom in patterns of hinge formation.

Another mathematical model employs a combination of the elasto-plastic rigid frame and the shear-wall. The use of this or any other model is, of course, based on the physical characteristics of thebuilding being analyzed. One variation of this combination is the use of an elastic rigid frame in combination with an elastic-perfectly plastic shear-wall (see reference B6). Construction details will also dictate whether the resistance of non-structural elements should be included in the model; generally the inclusion of such resistance when computing the complete response characteristics is impractical.

The choice of any particular mathematical model is dependent upon the purpose of the analysis, the characteristics of the basic structure and the analytical technique being used. Analytical tech niques will be discussed later in this lecture. The present discussion is concerned with the complexity of the various models insofar as the determination of the structural resistance and the applicability to certain types of structures.

It is evident that the shear-wall model is much simpler in concept than the elasto-plastic rigid frame. An analogy which is useful is the consideration of equivalent multiple mass and spring oscillators. The shear-wall model is analogous to a system in which any two adjacent masses are connected by only one spring. The resisting force at each mass is, therefore, only a function of the deformations of the two adjacent springs. By comparison the elasto-plastic rigid frame is analogous to a spring-mass system in which every mass is connected with every other mass by separate springs. In this case the resisting force at any mass is a function of the deformations of all the inter-connecting springs, i.e., a function of the displacements of all the other masses. Mathematically, the problem is the determination of the stiffness and flexibility matrices relating the resisting forces to the storey displacements and vice versa. For the shear-wall these matrices consist of only the three central diagonals and the elements can be determined by one or two simple calculations. For the elasto-plastic frame, these matrices usually contain all non-zero elements which must be determined by applying advanced methods of structural analysis. For elastic frames this analysis would only have to be made at the beginning of each problem so that the additional complexity is not of too much concern. However, when the frame is elastic-plastic, this analysis must be made each time a hinge forms and this increases the complexity of the situation enormously. When yielding occurs in any storey of the shear-wall structure, the inter-floor shear remains constant and the change in the stiffness matrix occurs only in one or two elements. It is for this reason that the shear-wall model is used wherever possible and the elasto-plastic rigid frame is generally used only by researchers.

The premise of the shear-wall model is that the floor systems are considered rigid compared to the flexible columns. This is usually valid in the cases where the floors are integral with the beam and girder systems and contribute to the strength of the frame whereas the wall systems are either panelled, hung or masonry and are not considered to significantly increase the column stiffnesses. Little investigation has been made into the validity of the shear-wall model in situations where the strength characteristics are not so clearly defined. Several investigators have included the non-structural resistance of masonry walls in determining the general effect on overall resistance. In cases of actual shear walls existing in buildings, the strength of the walls is included in calculating the properties of the shear wall, which is then combined with the structural frame in the determination of the overall resistance.

METHODS OF ANALYSIS

The basic problem in structural dynamics is to determine the response of the system to given dynamic loading (in this case, due to earthquake) or failing this to be able to estimate the maximum deformations and hence the amount of damage to be expected. Previous lecturers have discussed the methods of analysis for elastic systems and have shown that the method of modal analysis is perhaps the most useful. Because of the changing characteristics of the inelastic system, the method of modal analysis is usually not possible and most solutions are obtained by using some form of numerical integration of the equations of motion. It is the purpose of this section to discuss briefly some of the methods of analysis which have been used so that the engineer can have some idea of the different approaches which are available.

G. V. Berg (references B2, B5 and B6) has presented a method for the analysis of multi-storey elasto-plastic rigid frames. The structural resistance is evaluated by using the elastic equations at each time interval and superimposing linear "corrector" solutions for each point at which a plastic hinge has formed. The plastic hinge rotations at each time interval are determined by an iteration to satisfy the prescribed constraints within specified limits. Both the Milne Predictor-Corrector and Runge-Kutta methods of numerical integration were used in this analysis. The linear "corrector" solutions are obtained by pre-calculation for all points at which plastic hinges are expected to form.

J. A. Blume (references B10, B11 and B12) has presented a "Reserve Energy Technique" which is not a method of determining the complete response of a structure but which does provide an excellent means of estimating maximum deformation and damage to be expected due to a given earthquake. The bases of this technique are as follows:

(a) A consideration of the energy "balance including input energy loss, strain energy and energy "feed-back".

(b) A consideration of the force-deformation characteristics of the structure, the energy capacity under repeated excursions into the plastic range, and an estimate of the amount of energy fed back into the ground.

(c) A consideration of the energy input based upon knowledge of the natural period, the elastic acceleration spectra for earthquakes of given intensity and the reduction factor to be applied when using elastic spectra for inelastic situations.

The method of analysis is to estimate by trial and error the deformation at a position of energy balance. The evaluation of the structure is then made by means of a damage rating system based on the comparison of the energy balance deflection with the failure position, the no-damage position and the position of maximum permissible drift. A safety factor may be included in the calculations if desired.

R. W. Clough (reference C1) has described a method of analysis in which the moment-curvature characteristics for each member in the frame can be bilinear. The integration is done on a step-by-step basis with the assumption that the resistance function remains linear throughout each time interval. Changes in resistance due to plastic deformation are computed at the end of each time interval and used for the succeeding interval. The numerical method is based on the assumption that the acceleration varies linearly during the interval of integration.

A method of analysis for elasto-plastic rigid frames (references H1, H2 and H3) developed by the author is based upon the extension of the conjugate beam concept to elastic-plastic analysis. Each plastic hinge rotation is represented by a concentrated force on the conjugate of the rigid frame. The stiffness and flexibility matrices are then evaluated for each set of circumstances as it arises in the dynamic response problem. The method of numerical integration is a single step procedure which assumes a linear variation of acceleration and velocity over the time interval of integration.

R. K. Wen (references W2 and W3) has presented a method for the analysis of rigid frames whose members have general inelastic moment-curvature characteristics. This method is based on a "lumped-mass, lumped-flexibility" approach to determine the resistance of the frame at any position. A standard type of numerical integration was used.

A large number of papers (including references B7, B14, H4, K1, P1, P2 and S3) have used the shear building as a mathematical model. The differences between the various approaches are primarily in the form of numerical integration and in the variation of the yield shear force along the height of the frame. G. N. Bycroft (reference B14) describes an analysis using an analog computer to determine the response in which the elasto-plastic characteristic is provided by diode limited integrators. Several investigators (references K1 and S3) have used a bilinear form of shear force-deformation relationship, as shown in Figure 2 (d).

A number of other variations of the above methods could be described, but these represent a cross-section of such approaches. By consulting the appropriate reference publication, further information can be obtained on any particular method of analysis. It should be realized that computing the dynamic response of inelastic structural systems is a complex problem, regardless of the mathematical model or method of analysis to be used. This is why the determination of certain general characteristics of such systems is of such importance. The following section deals with such general characteristics.

GENERAL OBSERVATIONS ON THE EFFECTS OF INELASTIC ACTION

A large number of investigators have studied the behavior of particular inelastic structures when subjected to strong-motion earthquakes. Most investigations have dealt with the behavior of the single-storey shear frame model, i.e., a single mass oscillator. Such a system is simple enough so that the effects of parameters other than those related to inelastic behavior can often be eliminated.

Figure 6, taken from reference B10, shows typical elastic and elastic-plastic response curves for a single storey frame subjected to earthquake ground motion. Figure 7 (from reference VI) shows a comparison of the maximum relative displacements of elastic and elastic-plastic systems as a function of the natural period. Figures 8 and 9 (from reference VI) show the acceleration spectra for elastic and elastic-plastic systems and illustrate the effects of both damping and plastic deformation.

The figures referred to above illustrate typical response characteristics of simple systems subjected to strong-motion earthquakes. A number of observations can be made which are confirmed by the conclusions of many investigators. The most general conclusion is that the maximum dynamic elasto-plastic response of a structure decreases with a decrease in yield strength, i.e., an increase in ductility factor, until this strength reaches some optimum value at which point a further decrease in strength results in an increase in dynamic response. This general observation is shared by references B3, B10, P1 and V1. The position of this optimum yield strength varies but it is generally agreed that this occurs for values of ductility factor from about 10 to 20. Indications are that most structural configurations fabricated of ductile materials have a maximum useful ductility factor of about 8 (see reference B10); therefore, under most design conditions the effect of yielding in the practical range is to reduce total deformation. Further observations (see reference A1) have been made indicating that the above phenomenon is only generally true for a range of natural period above a minimum which is usually around 1.0 seconds. That this is true can be seen by a close examination of Figure 7.

Another general observation is that the effect of inelastic action on the maximum dynamic response is of the same order as the effect of viscous damping and both are roughly additive. The authors of references B6, B10, and P1 are in agreement on this point. An examination of Figures 8 and 9 shows that this effect is also true for the spectral acceleration. These figures also illustrate how the lateral design coefficient can be reduced by allowing a certain amount of plastic deformation.

G. V. Berg, in reference B3, observes that the total input energy is generally reduced when yielding is permitted. This is in agreement with previous remarks regarding the decrease of maximum deformation when yielding is permitted.

Several detailed studies have been made of the dynamic response of multi-storey frames (see in particular references B6 and P1). A typical set of response curves for elastic and elastic-plastic cases are shown in Figures 10 and 11 (from reference P1). It is characteristic of the elastic-plastic response of such systems that the period of vibration remains about the same as for the response and that the vibration is essentially elastic about a moving equilibrium position which is constantly shifting away from the zero position as inelastic deformation progresses. The same observation may be made about single degree-of-freedom systems, but the effect is less obvious in Figure 6. Another observation is that the maximum deformation (for the case shown in Figure 11) is larger than the elastic deformation but that the amplitude of oscillation is much smaller.

It has been the purpose of this section to describe some of the more general observations with the view of establishing some intuitive basis for judging the effects of inelastic behavior. This is especially valuable in cases where any type of detailed analytical investigation, i.e., computing probable response curves to prior earthquakes, is not possible. It should be mentioned that many of the observations are also valid for dynamic loadings other than those due to earthquakes, e.g., impulse and blast loading.

CONCLUSIONS

The general purpose of this lecture has been to describe the nature of structural inelasticity, the resulting mathematical models, the methods of analysis and the qualitative effects which this inelasticity can have on the response characteristics of structural systems. Due to the short time and space available, many of the details have had to be completely eliminated; however, the list of references which follow make it possible for these details to be obtained at the leisure of the reader.

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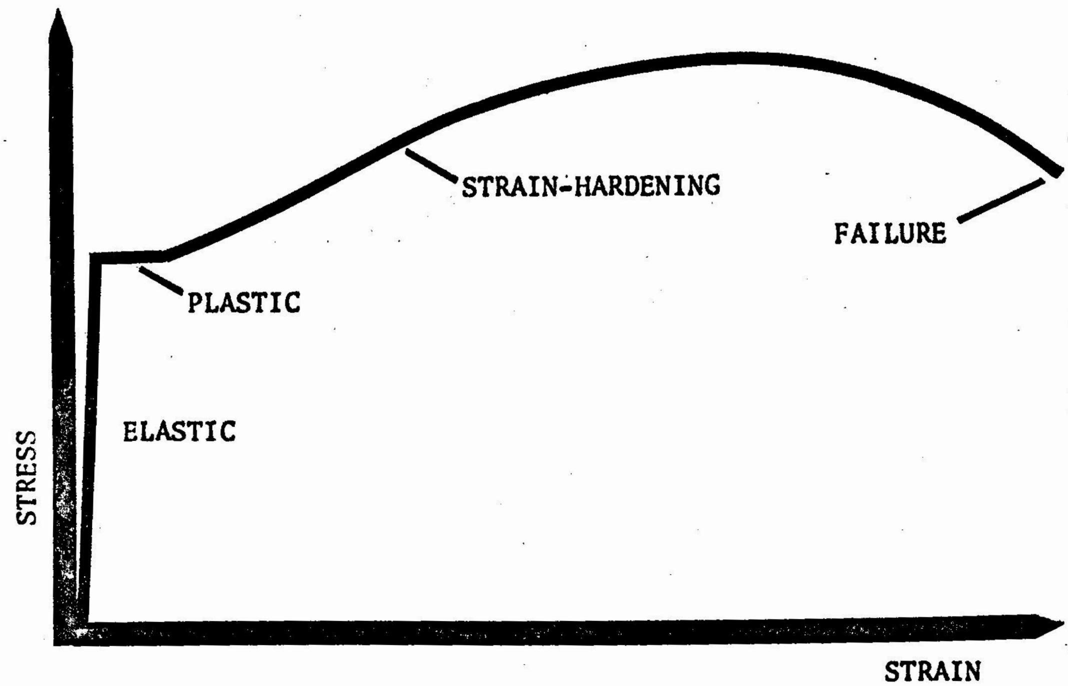
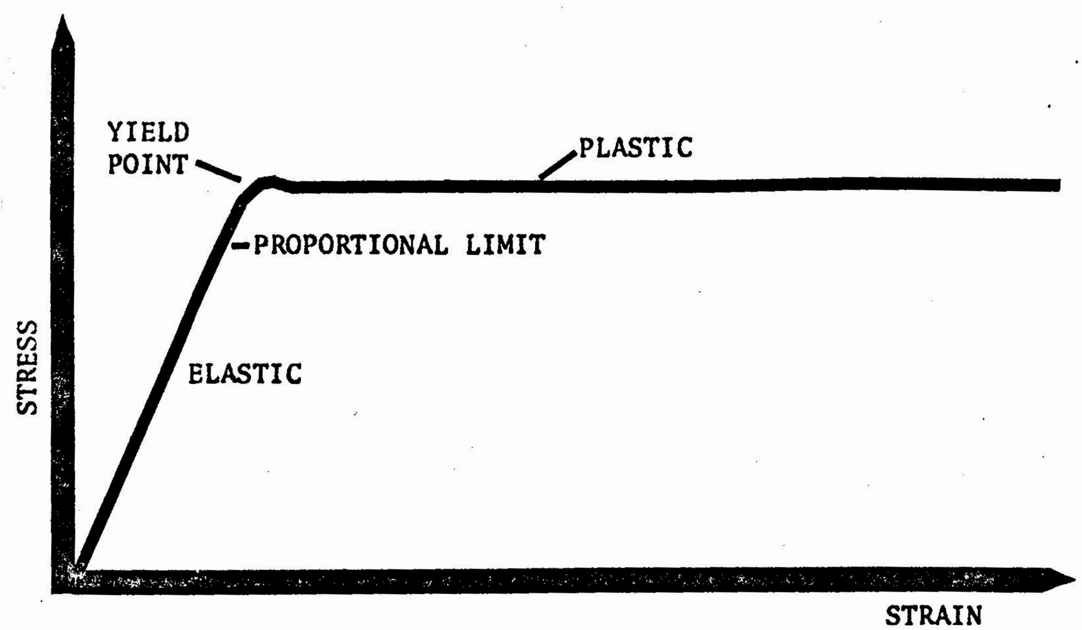
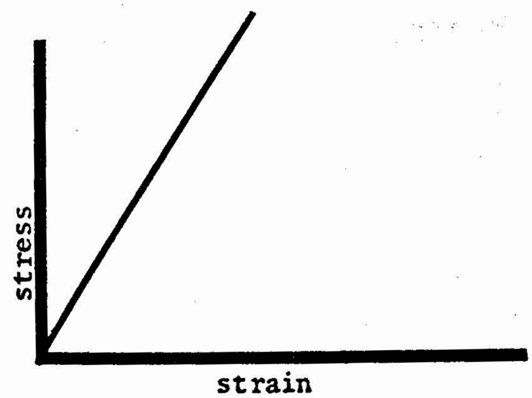


a) stress-strain curve to failure

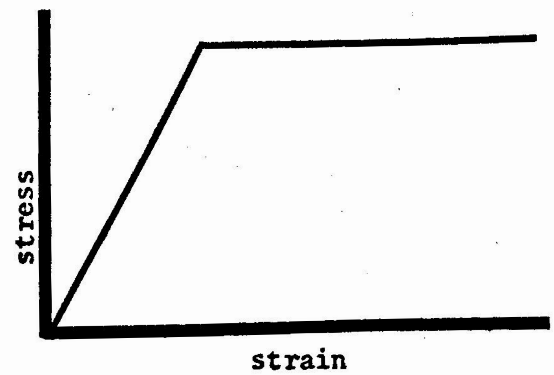


b) curve exaggerated in regions of lower strain

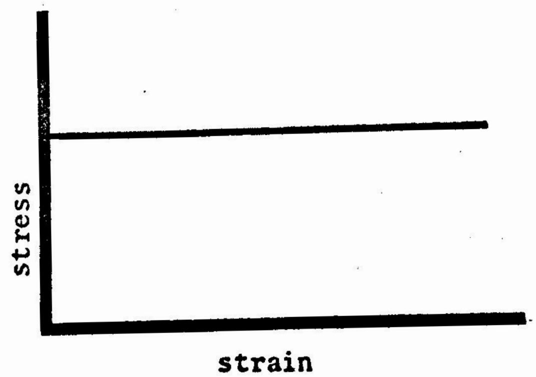
Figure 1: Stress-Strain Diagram for Mild Steel



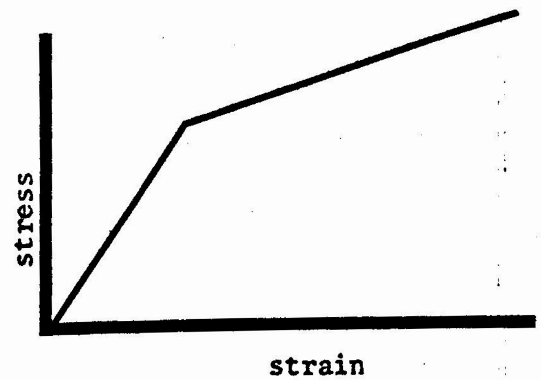
a) linear elastic



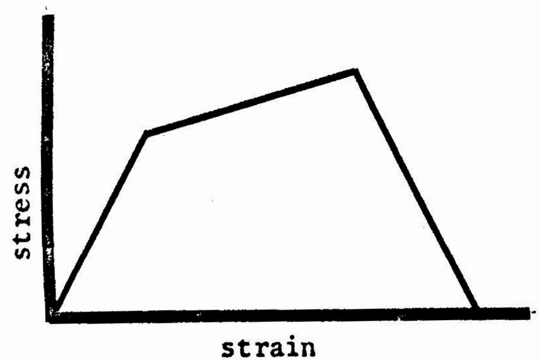
b) elastic-perfectly plastic



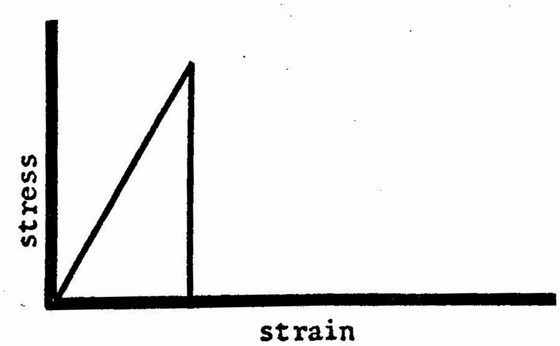
c) rigid-plastic



d) bi-linear elastic-plastic

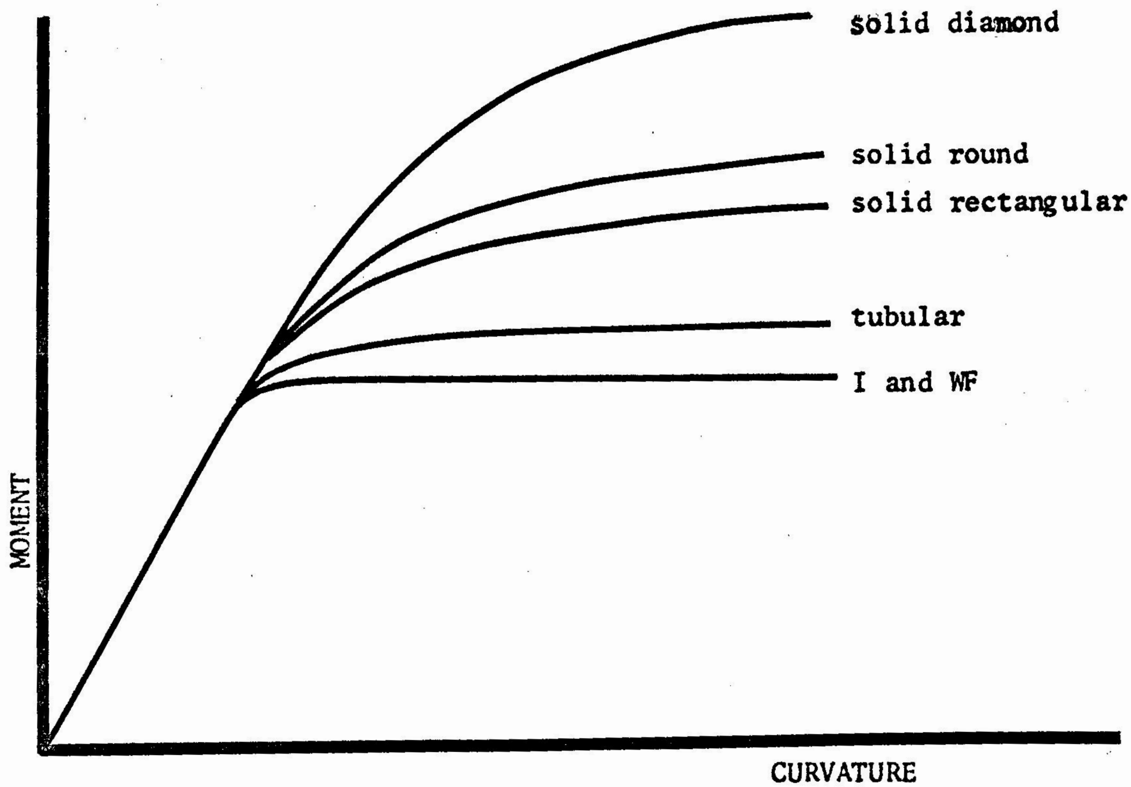


e) non-structural inelasticity

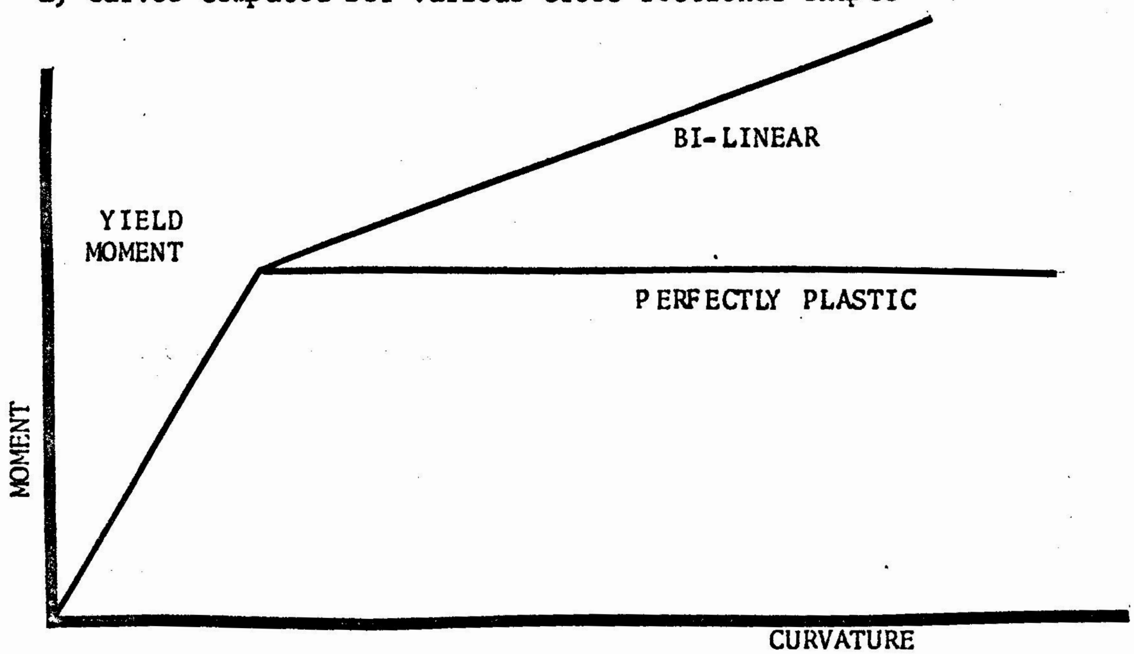


f) elastic-to-failure

Figure 2: Various Types of Stress-Strain Relationship

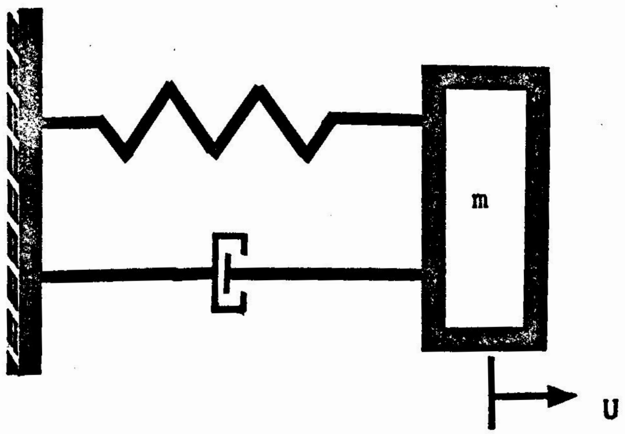


a) Curves computed for various cross-sectional shapes

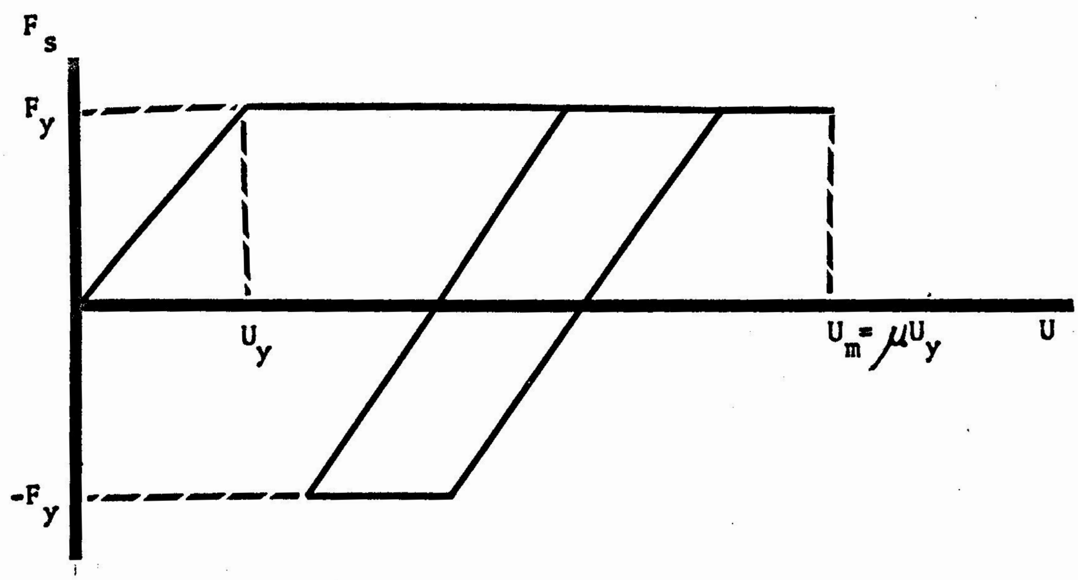


b) Idealized moment-curvature characteristics

Figure 3: Moment-Curvature Relationships (for materials having elastic-perfectly plastic stress-strain characteristics)

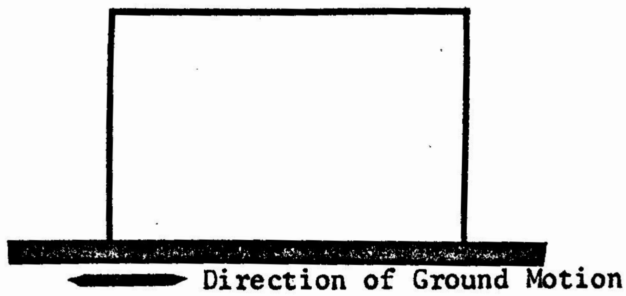


a) Single Degree of Freedom System

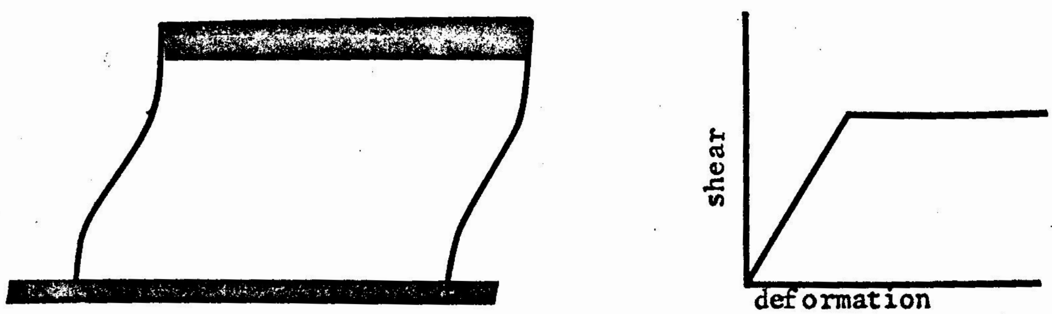


b) Elastic-Perfectly Plastic Spring Force

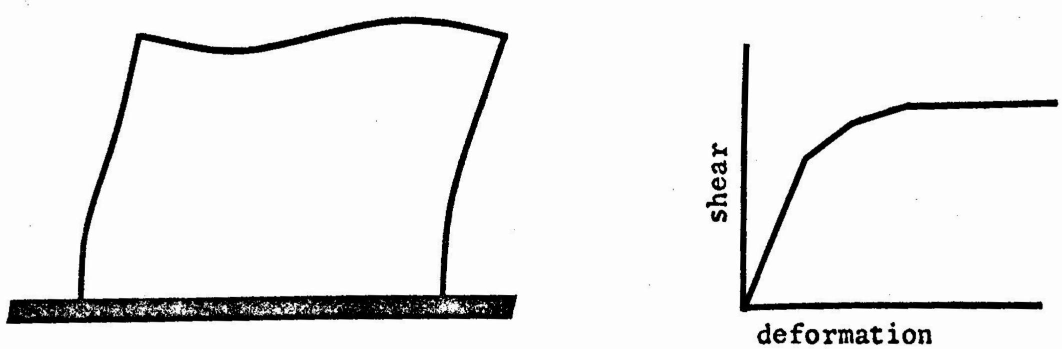
Figure 4: Single Mass Oscillator



a) basic frame



b) rigid floor model



c) flexible floor model

Figure 5: Single Storey Rigid Frame

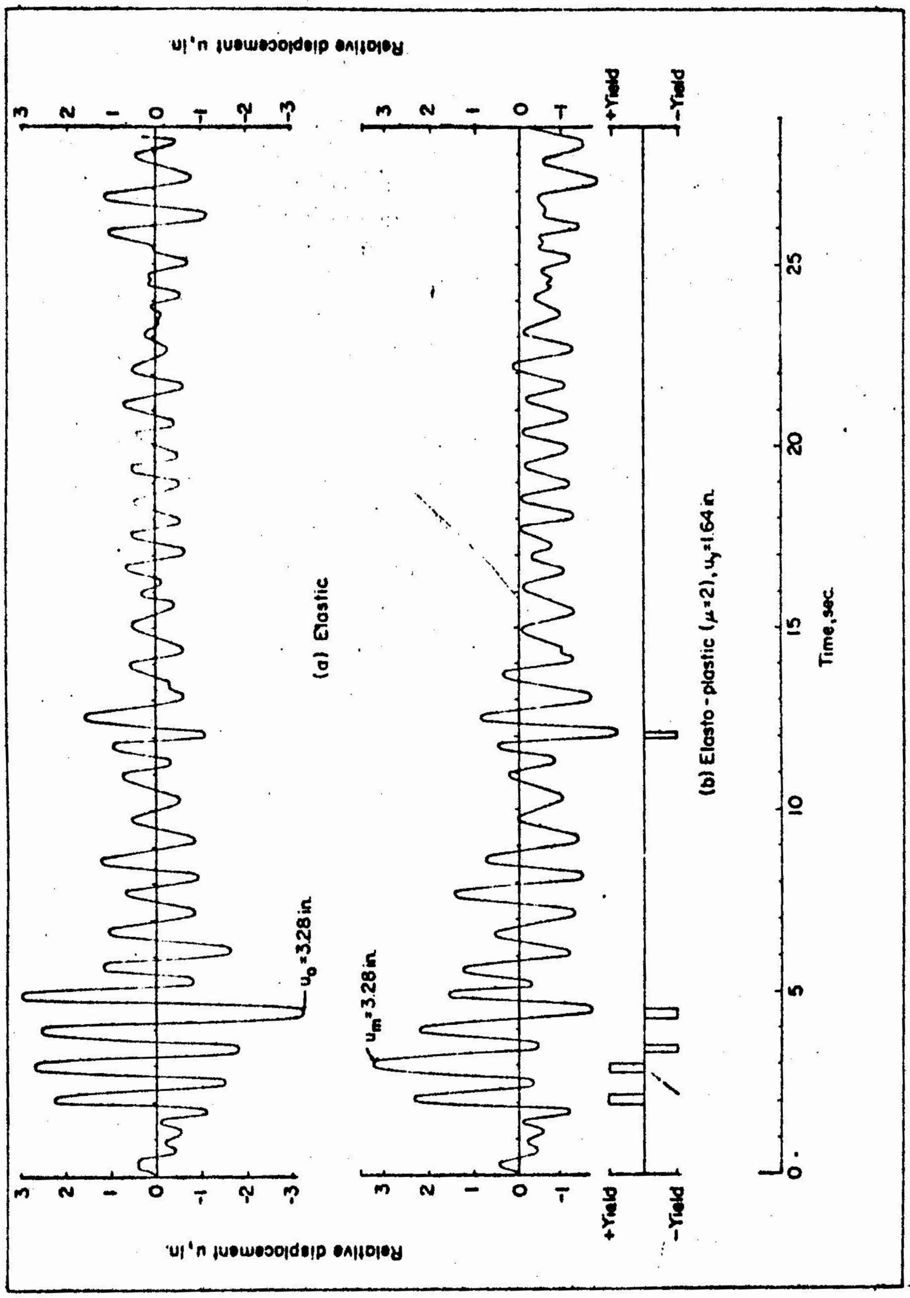


Fig. 1-4. Response of a system with $T = 1.0$ second, $\beta = 0.10$; 1940 El Centro, Calif., earthquake, N-S component.

Figure 6 - reprinted from reference B10

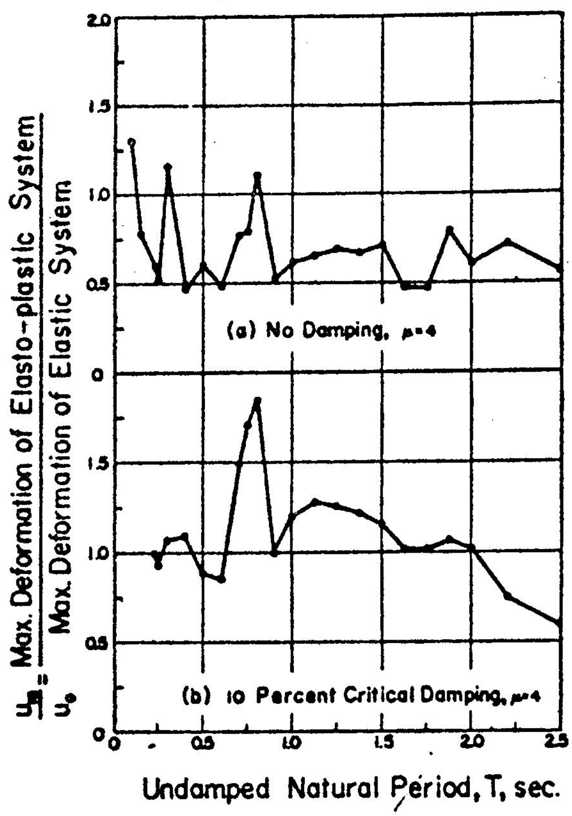


Fig. 5 Comparison of Maximum Relative Displacements of Elasto-Plastic and Elastic Systems as a Function of Natural Period -- El Centro Earthquake

Figure 7 - reprinted from reference V1

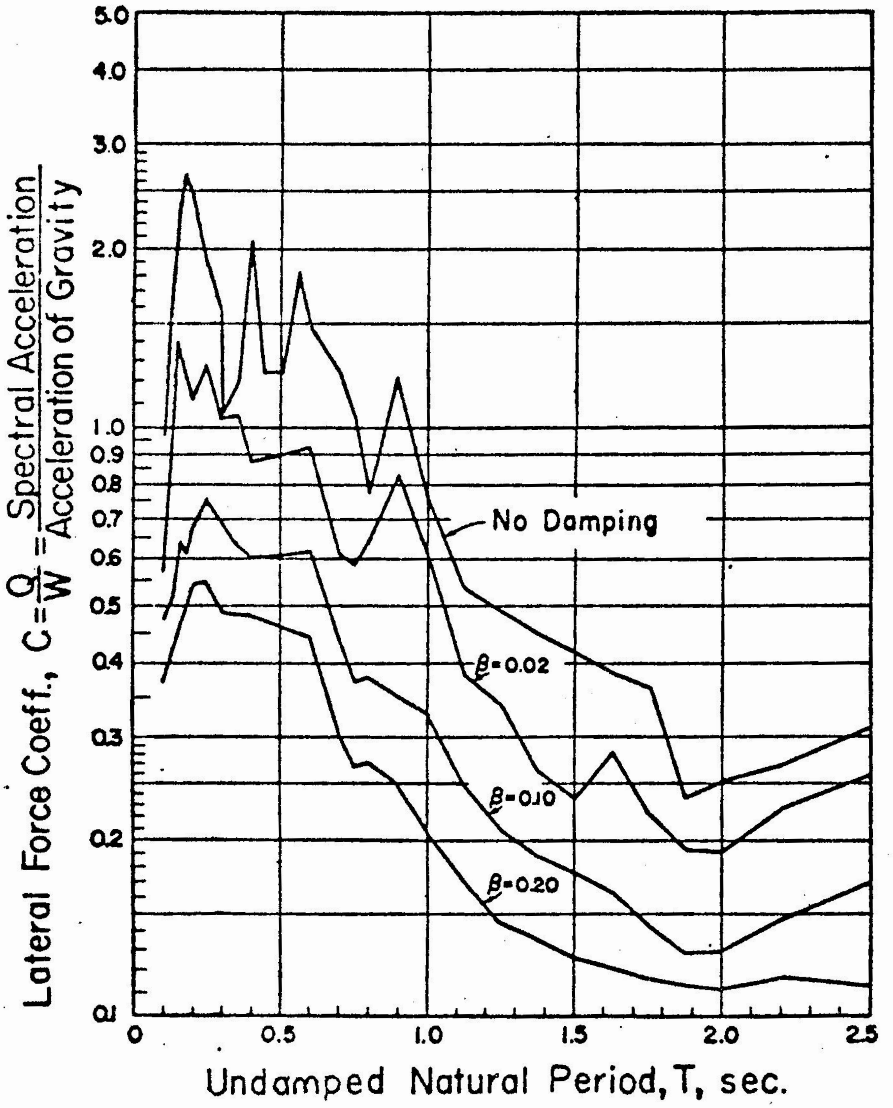


Fig. 8 Acceleration Spectra for Elastic Systems -- El Centro Earthquake

Figure 8 - reprinted from reference V1

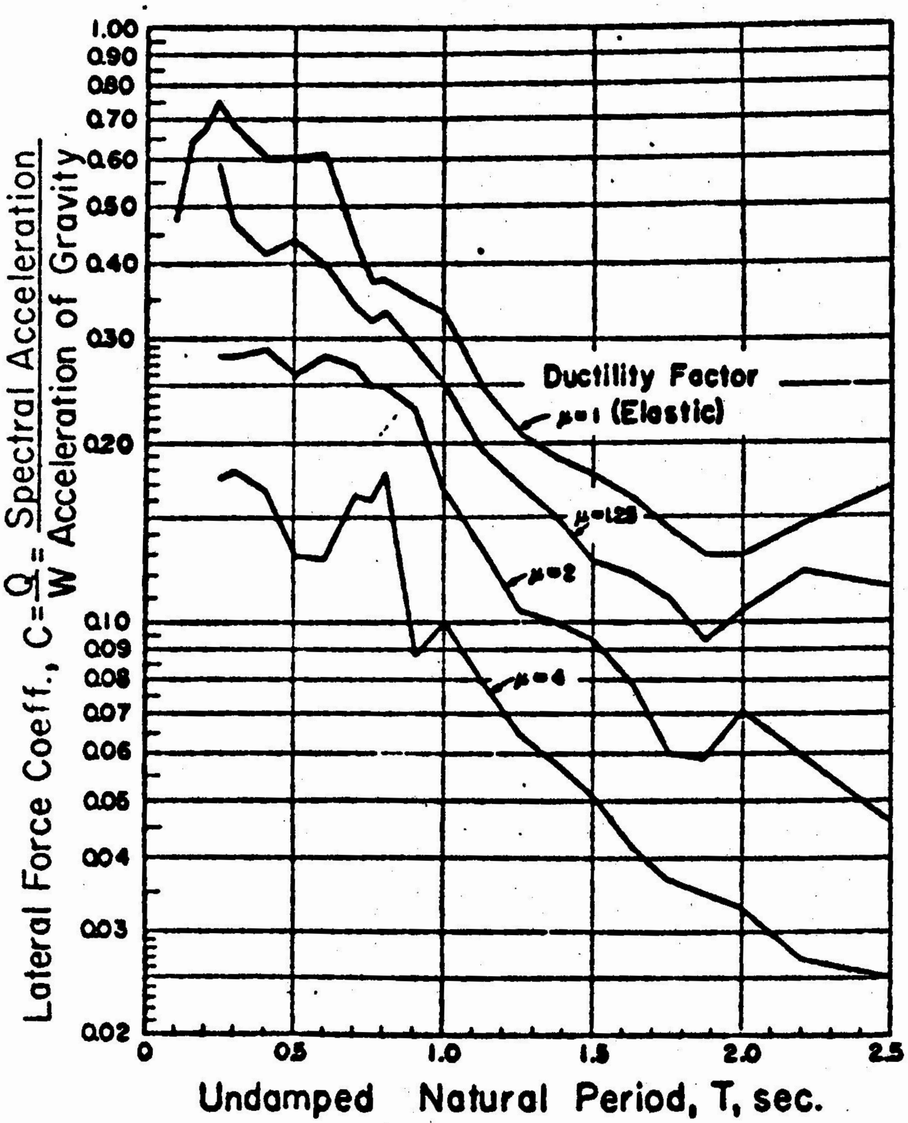


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Fig. 9 Acceleration Spectra for Elasto-Plastic Systems with 10 Percent Critical Damping -- El Centro Earthquake

Figure 9 - reprinted from reference V1

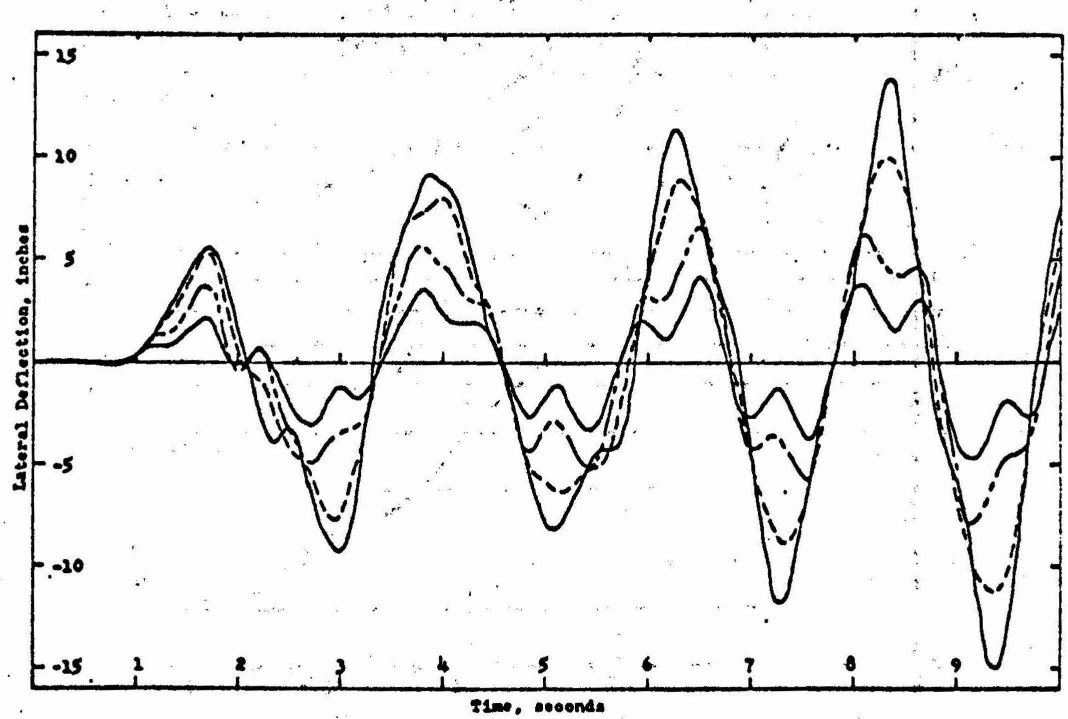

FIG. 9.—RESPONSE OF FOUR-STORY UNDAMPED ELASTIC FRAME, EL CENTRO EARTHQUAKE

Figure 10 - reprinted from reference P1

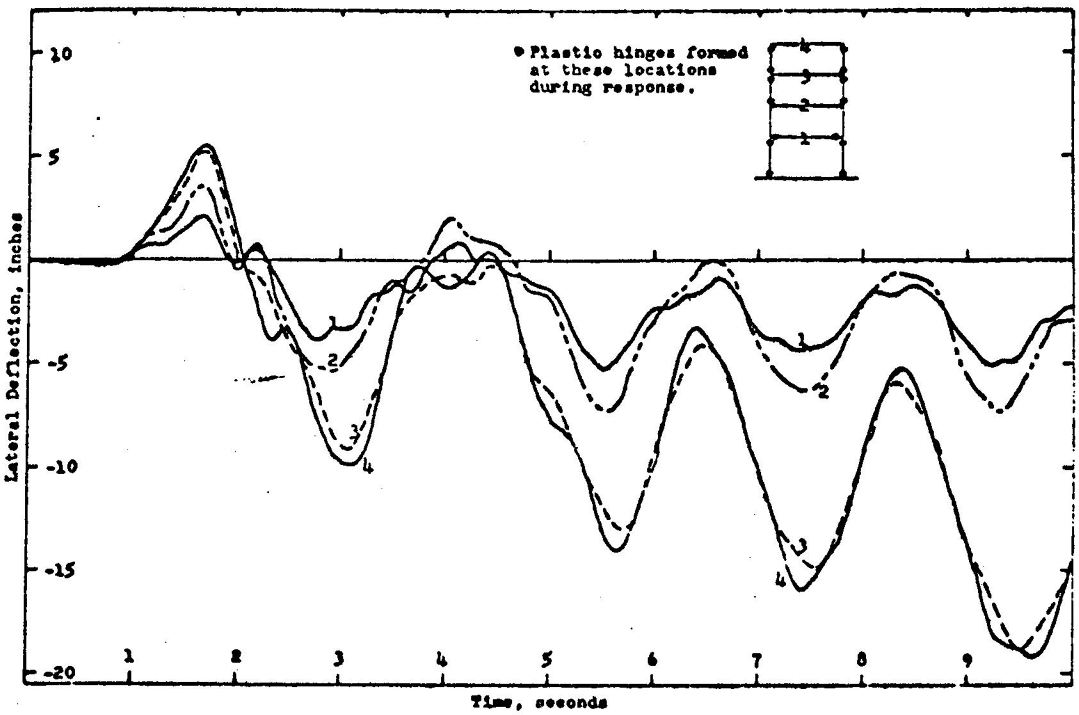

FIG. 10.—RESPONSE OF FOUR-STORY ELASTIC-PLASTIC FRAME, EL CENTRO EARTHQUAKE

Figure 11 - reprinted from reference P1